

研究简报

水的离子积 $pK_w = f(T, P)$ 计算公式

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本文在系统地研究了国内外水的离子积计算公式基础上, 根据化学热力学理论, 提出临界区和超临界区水的离子积 $pK_w = f(T, P)$ 计算公式。与 Sweeton 实验值比较, $0^\circ \sim 300^\circ \text{C}$ pK_w 平均偏差 0.008; $0^\circ \sim 300^\circ \text{C}$, 1—1000 bar pK_w 计算值接近 Marshall 公式的计算值, 但比它实用。同时, 可以导出 $pK_w = f(T)pK_w = f(T, \rho)$ 类型的计算公式。

关键词: 水的离子积 化学热力学 Gibbs自由能 热容

水的离子积 pK_w 是水的重要物理化学性质之一。F.H.Sweeton 用电导法测定 $0^\circ \sim 300^\circ \text{C}$ 水的 pK_w 值⁽¹⁾, W.L.Marshall 报道了超临界区域水的值测定方法⁽²⁾。国内外关于水的离子积计算公式可归纳为三类十一种公式:

(1) Dale.F.Taylor (1978)⁽³⁾

$$pK_w = -(\mu_{\text{H}^+}^\circ + \mu_{\text{OH}^-}^\circ - \mu_{\text{H}_2\text{O}}^\circ)/2.303RT \quad (1)$$

(2) Н.И.Еремкин (1968)⁽⁴⁾

$$\log K_w = -24.5 + 0.0349 T \quad (2)$$

(3) 傅培鑫(1982)⁽⁵⁾

$$\log K_w = \alpha_i + \beta_i T \quad (3)$$

(4) 陈国强(1984)⁽⁶⁾

$$\log K_w(T) = \log K_w(T_0) + (a - bt)tT^{-1} \quad (4)$$

(5) F.H.Sweeton (1974)⁽¹⁾

$$\log K_w(T) = 21.647 - 4239.4 T^{-1} - 3.7546 \ln T \quad (5)$$

(6) 周培根(1984)⁽⁷⁾

$$pK_w(T) = A + B \ln T + CT + DT^{-1} + ET^{-2} \quad (6)$$

(7) W.B.Holzappel (1969)⁽¹⁾

$$\log K_w = -3.58 - 3108T^{-1} + (12.4 + 5.0\rho) \log \rho \quad (7)$$

(8) G.J.Bignold (1971)⁽¹⁾

$$\log K_w = -3.8418 - 3030.2T^{-1} + (12.4 + 5.0\rho) \log \rho \quad (8)$$

(9) A.S.Quist (1970)⁽¹⁾

$$\log K_w = -3.74 - 3050T^{-1} + 14.8 \log \rho \quad (9)$$

(10) A.S.Quist (1970)⁽⁸⁾

$$\log K_w = -3.05 - 3050T^{-1} + 16.8 \log \rho \quad (10)$$

(11) W.L.Marshall (1981)⁽²⁾

$$\log K_w = \log K_w^* + K \log \rho \quad (11)$$

然而, $\log K_w = f(G)$ 、 $\log K_w = f(T)$ 计算公式仅适用于临界区域内水的 $\log K_w$ 计算, $\log K_w = f(T, \rho)$ 适用范围广些, 但因高温高压下液态水的密度数据缺乏或者测定困难, 所以实际使用时受到限制。同时, (1)~(11)式中的常数项都是根据实验数据用最小二乘法求得的经验参数, 缺乏确切的物理化学概念。因此, 本文根据化学热力学理论, 提出 $\log K_w = f(T, P)$ 类型计算公式。

我们研究了 $H_2O \rightleftharpoons H^+ + OH^-$ 体系热力学平衡条件下物态方程 $f(T, P, V) = 0$, 得到Gibbs自由能、体积变化量及其 ΔV_T^p 的关联式:

$$\Delta G_T^p = \Delta G_T^o + \int_1^p \Delta V_T^p dP$$

$$\Delta G_T^o = \Delta H_0 + (\Delta a - \Delta S_0)T - \Delta a T \ln T - \frac{1}{2} \Delta b T^2 - \frac{1}{2} \Delta c T^{-1}$$

$$\Delta V_T^p \approx \Delta V_T^o = \Delta V^o - aT^b$$

$$\text{于是, } \log K_w(T, P) = \Delta G_T^p / 2.303 RT = \log K_w(T, P_0) + \log K_w'(T, P) \quad (12)$$

$$\log K_w(T, P_0) = \Delta G_T^o / 2.303 RT = A + B \ln T + CT + DT^{-1} + ET^2 \quad (13)$$

$$\log K_w'(T, P) = \Delta V_T^p (P - 1) / 2.303 RT = (FT^{-1} - HT^m) (P - 1) \quad (14)$$

(12)~(14)式中常数项全可从 $H_2O \rightleftharpoons H^+ + OH^-$ 体系的化学热力学数据求出:

$$A = (\Delta a - \Delta S_0) / 2.303 R = -20.695, B = -\Delta a / 2.303 R = 2.764,$$

$$C = -\frac{1}{2} \Delta b / 2.303 R = 10.37 \times 10^{-3}, D = \Delta H_0 / 2.303 R = 4793,$$

$$E = -\frac{1}{2} \Delta c / 2.303 R = -0.189 \times 10^5, F = \Delta V^o / 2.303 R = -0.1167,$$

$$H = a / 2.303 R = 6.6636 \times 10^{-23}, m = b - 1 = 6.9693,$$

$$R = 8.314 \text{ J} \cdot \text{K}^{-1} \cdot \text{mole}^{-1} \text{ 或 } 83.1437 \text{ cm}^3 \cdot \text{bar} \cdot \text{K}^{-1} \cdot \text{mole}^{-1}$$

表1 水的离子积 pK_w 实验值和计算值 ($0 \sim 300^\circ\text{C}$, 饱和蒸汽压)
 Table 1 Experimental and Calculated Value for Water Ion Product pK_w (Under $0^\circ - 300^\circ\text{C}$ and Steam Pressure)

tempe- rature °C	experimental value of pK_w sweeton ⁽¹⁾ (1974)	$pK_w = -\log K_w$ calculated value by equation												
		$pK_w = f(T)$	$pK_w = f(T, \rho)$											
		$pK_w = f(G)$	equ. (1) ⁽¹⁾ Taylor (1978)	equ. (2) ⁽⁴⁾ Epe Fu (1968)	equ. (3) ⁽³⁾ Fu Peixing (1982)	equ. (4) ⁽⁶⁾ Chen Guoqiang (1984)	equ. (5) ⁽¹⁾ Swee ton (1974)	equ. (6) ⁽⁷⁾ Zhou peigeng (1984)	equ. (7) ⁽¹⁾ Holza ptel (1969)	equ. (8) ⁽¹⁾ Bignold (1971)	equ. (9) ⁽¹⁾ Quist (1970)	equ. (10) ⁽⁸⁾ Quist (1970)	equ. (11) ⁽²⁾ Marshall (1981)	$pK_w = f(T, \rho)$ equ. (13) author in this paper
0	14.841±0.009	—	14.97	14.98	14.943	14.936	14.945	14.96	14.93	14.91	14.22	14.938	14.938	14.938
25	13.893±0.009	13.994	14.09	14.08	13.996	13.964	13.994	14.02	14.02	13.99	13.30	13.995	13.995	14.008
50	13.272±0.006	13.268	13.22	13.18	13.263	13.167	13.271	13.29	13.31	13.26	12.58	13.275	13.275	13.278
75	12.709±0.006	12.700	12.35	12.68	12.694	12.504	12.708	12.70	12.73	12.66	12.00	12.712	12.712	12.703
100	12.264±0.009	12.258	11.48	12.18	12.280	11.949	12.263	12.23	12.28	12.18	11.53	12.265	12.265	12.252
125	11.814±0.009	11.917	10.60	11.97	11.935	11.479	11.913	11.86	11.92	11.80	11.17	11.912	11.912	11.901
150	11.642±0.012	11.657	9.73	11.75	11.699	11.079	11.642	11.56	11.64	11.50	10.89	11.638	11.638	11.630
175	11.441±0.012	11.465	8.86	11.54	11.534	10.735	11.441	11.36	11.44	11.29	10.69	11.432	11.432	11.428
200	11.302±0.012	11.329	7.99	11.33	11.435	10.439	11.302	11.20	11.29	11.12	10.55	11.289	11.289	11.283
225	11.222±0.012	11.241	7.11	11.26	11.389	10.182	11.221	11.16	11.26	11.06	10.53	11.208	11.208	11.184
250	11.196±0.015	11.191	6.24	11.19	11.388	9.960	11.196	11.12	11.23	11.01	10.52	11.191	11.191	11.125
275	11.224±0.027	11.176	5.37	11.12	11.421	9.766	11.223	11.20	11.32	11.08	10.64	11.251	11.251	11.101
300	11.301±0.045	11.189	4.50	11.05	11.494	9.596	11.301	11.37	11.50	11.26	10.87	11.406	11.406	11.109
average	0-100°C 0.008	0.005	0.264	0.066	0.008	0.132	0.002	0.022	0.024	0.034	0.711	0.002	0.002	0.008
deviation	0-300°C 0.014	0.023	2.473	0.079	0.072	0.667	0.001	0.049	0.040	0.098	0.687	0.015	0.015	0.04

本文提出的 $pK_w = f(T, P)$ 及综述的 $pK_w = f(G)$ 、 $pK_w = f(T)$ 、 $pK_w = f(T, \rho)$ 类型计算公式的 $0^\circ \sim 300^\circ\text{C}$ pK_w 计算值及其与 Sweeton 实验值比较的平均偏差列于 (表 1), 除 (2)、(5)、(10) 式外, 其余 pK_w 计算公式均是满意的。本文作者提出的 (3) 式改进了苏联 Еремин (2) 式, 而且, 比 (4)、(1) 式用在水的离解自由能、 pH (中性点) 计算、 pH_t 换算以及盐类水解反应的研究中更为方便^(5,9)。

表 2 $0^\circ \sim 300^\circ\text{C}$, 1-1000 bar 水的离子积 pK_w 计算值

Table 2 Calculated Value for Water Ion Product pK_w Under $0^\circ \sim 300^\circ\text{C}$ and 1-1000 bar

colcd. by equation pressure (bar)		temperature °C								
		0	25	50	75	100	150	200	250	300
satd. vapor	(11)	14.938	13.995	13.275	12.712	12.265	11.638	11.289	11.191	11.406
	(12)	14.938	14.008	13.278	12.703	12.252	11.630	11.283	11.125	11.109
250	(11)	14.83	13.90	13.19	12.63	12.18	11.54	11.16	11.01	11.14
	(12)	14.83	13.91	13.18	12.61	12.16	11.53	11.15	10.92	10.78
500	(11)	14.72	13.82	13.11	12.55	12.10	11.45	11.05	10.85	10.86
	(12)	14.72	13.81	13.09	12.52	12.07	11.43	11.01	10.72	10.45
750	(11)	14.62	13.73	13.04	12.48	12.03	11.36	10.95	10.72	10.66
	(12)	14.61	13.71	12.99	12.43	11.98	11.32	10.88	10.52	10.12
1000	(11)	14.53	13.66	12.96	12.41	11.96	11.29	10.86	10.60	10.50
	(12)	14.50	13.61	12.90	12.33	11.88	11.22	10.74	10.31	9.79

本文提出的 $pK_w = f(T, P)$ 计算公式优于其他公式, 适用于临界区和超临界区 pK_w 计算, 同 Sweeton 实验值比较, $0^\circ \sim 300^\circ\text{C}$ pK_w 平均偏差 0.008; 超临界区 1—1000 bar, $0^\circ \sim 300^\circ\text{C}$ pK_w 计算值接近 Marshall 的公式计算值 (表 2), 克服了 $pK_w = f(T, \rho)$ 公式中 ρ 值测定的困难。同时, 可以导出 $pK_w = f(T)$ 、 $pK_w = f(T, \rho)$ 类型公式。例如, (13) 式类似周培根的 (6) 式; 由

$$(\partial \ln K_w / \partial \ln \rho)_T = \left(\frac{1}{\beta'} \right) (\partial \ln K_w / \partial P)_T$$

和 $(\partial \ln K_w / \partial P)_T = K\beta' = \Delta V_T^\circ / RT$ 关系方程⁽¹⁾⁽¹⁶⁾, 得出 $pK_w'(T, P) = \Delta V_T^\circ (P-1) / 2.303 RT = -K \log \rho$, 当 $\Delta G_T^\circ = A_0 + C_0 T$ 时 $pK_w(T, P) = A + DT^{-1} - K \log \rho$ 类似 W.B. Holzapfel, G.J. Bignold, A.S. Quist 的 (7) 式、(8) 式、(9) 式、(10) 式。

$$\text{若} \quad \Delta G_T^\circ = \Delta H_0 + (\Delta a - \Delta S_0)T - \frac{1}{2} \Delta b T^2 - \frac{1}{2} \Delta c T^{-1} - \frac{1}{2} \Delta d T^{-2},$$

$$pK_w(T, P_0) = A + DT^{-1} + ET^{-2} + FT^{-3}, \text{ 则}$$

$$pK_w(T, P) = pK_w(T, P_0) + pK_w'(T, P) = pK_w(T, P_0) - K \log \rho$$

类似 M.L. Marshall 的 (11) 式。因此, 本文提出的 $pK_w = f(T, P)$ 公式是普遍适用的水的离子积计算公式。

参 考 文 献

- [1] Sweeton, F.H., Mesmer, R.E., Baes, C.F., *J.Solu.Chem.*, 3(3), 191~214(1974).
- [2] Marshall, W.L., Franck, E.U., *J.Phys.Chem.Ref.Data*, 10(2), 295—304 (1981).
- [3] Taylor, D.F., *J.Electrochem.Soc.*, 125(5), 808—812(1978).
- [4] Еремнн, Н.И., Цветные.Металлы, (12), 52—56(1968).
- [5] 傅培鑫, 科学通报(中文版), (11), 74(1982); (英文版), (11), 1245(1982).
- [6] 陈国强、相进元 化学通报, (12), 38-42(1984).
- [7] 周培根, 化学学报, (10), 1092-1093(1984).
- [8] Quist, A.S., *J.Phys.Chem.*, 74(18), 3396-3402(1970).
- [9] 傅培鑫, 无机化学, 2(2), 103-107(1986).
- [10] Marshall, W.L., *J.Phys.Chem.*, 74(2), 346-355(1970).

A PRACTICAL FORMULA FOR WATER IONIC PRODUCT $pK_w=f(T,P)$

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In this paper an adapted and more practical formula for the water ionic product $pK_w=f(T,P)$ in critical and above-critical zones has been put forward, which is based on the systematical study no the existing formulae and on the chemical thermodynamical theory. Compared with Sweeton's experimental pK_w value under 0—300℃, the average deviation of the pK_w value obtained by the adapted formula is 0.008, which is close to that by Marshall's formula under 0—300℃ and 1—1000 bar. From this formula, other relevant ones for $pK_w=f(T)$, $pK_w=f(T,p)$ can also be derived.

Keywords water ionic product chemical thermodynamics Gibbs
free energy thermal capacitance